Program Stream Stochastic Processes

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Benjamin Robinson: **Bicausal optimal transport for SDEs with irregular coefficients**
Helena Katharina Kremp: **Higher order approximation for nonlinear SPDEs with additive space-time white noise**

Plenary Stochastic Processes:
Gonçalo dos Reis: **High order splitting methods for stochastic differential equations**

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Luca Pelizzari: **Rough PDEs for local stochastic volatility models**
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Talks III:
Peter Friz: **Signatures, developments, smooth rough paths and triviality of paths with polynomial log-signature**
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Fabio Bugini: **Continuity and differentiability of rough stochastic differential equations with respect to a parameter**

Abstracts

Speaker (Plenary):
Gonçalo dos Reis

Title:
High order splitting methods for stochastic differential equations

Abstract:
In this talk, we will discuss how ideas from rough path theory can be leveraged to develop high order numerical methods for SDEs. To motivate our approach, we consider what happens when the Brownian motion driving an SDE is replaced by a piecewise linear path. We show that this procedure transforms the SDE into a sequence of ODEs – which can then be discretized using an appropriate ODE solver. Moreover, to achieve a high accuracy, we construct these piecewise linear paths to match certain “iterated” integrals of the Brownian motion. At the same time, the ODE sequences obtained from this path-based approach can be interpreted as a splitting method, which neatly connects our work to the existing literature. For example, we show that the well-known Strang splitting falls under this framework and can be modified to give an improved convergence rate. We will conclude the talk with a couple of examples, demonstrating the flexibility and convergence properties of our methodology.

This is joint work with James Foster and Calum Strange.

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Speaker:
Luca Pelizzari

Title:
Rough PDEs for local stochastic volatility models

Abstract:
In this talk, we introduce a novel pricing methodology in general, possibly non-Markovian local stochastic volatility (LSV) models. We observe that by conditioning the LSV dynamics on the Brownian motion that drives the volatility, one obtains a time-inhomogeneous Markov process. Using tools from rough path theory, we describe how to precisely understand the conditional LSV dynamics and reveal their Markovian nature. The latter allows us to connect the conditional dynamics to so-called rough partial differential equations (RPDEs), through a Feynman-Kac type of formula. In terms of European pricing, conditional on realizations of one Brownian motion, we can compute conditional option prices by solving the corresponding linear RPDEs, and then average over all samples to find unconditional prices. Our approach depends only minimally on the specification of the volatility, making it applicable for a wide range of classical and rough LSV models, and it establishes a PDE pricing method for non-Markovian models.

The talk is based on joint work with Peter Bank, Christian Bayer and Peter Friz.

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Speaker:
Nikolas Tapia

Title:
Branched Itô Formula and natural Itô-Stratonovich isomorphism

Abstract:
Branched rough paths define integration theories that may fail to satisfy the integration by parts identity. The projection of the Connes-Kreimer Hopf algebra H_{CK} onto its primitive elements defined by Broadhurst-Kreimer and Foissy, allows us to view H_{CK} as a commutative B_\infty-algebra and thus to write an explicit change-of-variable formula for solutions to rough differential equations (RDEs), which restricts to the well-known Itô formula for semimartingales. When compared with Kelly's approach using bracket extensions, this formula has the advantage of only depending on internal structure. We proceed to define an isomorphism between H_{CK} and Sh(P) (the shuffle algebra over primitives), which we compare with the previous constructions of Hairer-Kelly and Boedihardjo-Chevyrev: while all three allow one to write branched RDEs as RDEs driven by geometric rough paths taking values in a larger space, the key feature of our isomorphism is that it is natural when H_{CK} and Sh(P) are viewed as covariant functors Vec to Hopf. Our

natural isomorphism extends Hoffman’s exponential for the quasi-shuffle algebra, and in particular the usual Itô-Stratonovich correction formula for semimartingales. Special emphasis is placed on the 1-dimensional case, in which certain rough path terms can be expressed as polynomials in the trace path indexed by P, which for semimartingales restrict to the well-known Kailath-Segall polynomials.

This talk is based on joint work with E. Ferrucci and C. Bellingeri.

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Speaker:
Helena Katharina Kremp

Title:
Higher order approximation for nonlinear SPDEs with additive space-time white noise

Abstract:
We consider strong approximations of 1+1-dimensional stochastic PDEs driven by additive space-time white noise. It has been long proposed (Davie-Gaines '01, Jentzen-Kloeden '08), as well as observed in simulations, that approximation schemes based on samples from the stochastic convolution, rather than from increments of the underlying Wiener processes, should achieve significantly higher convergence rates with respect to the temporal timestep. For a large class of nonlinearities, with possibly superlinear growth, we prove a temporal rate of (almost) 1, a major improvement on the rate 1/4 that is known to be optimal for schemes based on Wiener increments. The spatial rate remains (almost) 1/2 as it is standard in the literature.

The talk is based on a joint work with Ana Djurdjevac and Máté Gerencsér.

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Speaker:
Fabio Bugini

Title:
Continuity and differentiability of rough stochastic differential equations with respect to a parameter

Abstract:
We study rough stochastic differential equations (RSDEs), with coefficients and initial condition depending on a parameter. A RSDE is a stochastic differential equation simultaneously driven by a Brownian motion and a deterministic rough path. Our focus is to understand the dependence of solutions to such equations on the parameter. Specifically, we investigate under which assumptions on the coefficients we have continuity and/or differentiability in mean with respect to the parameter. As an application, we show how our theory can be useful in providing an existence-and-uniqueness result for a class of linear backward rough partial differential equations (PDEs). Using a rough Feynman-Kac formula,

we prove that solutions of such rough PDEs admit a functional integral representation in term of solutions of appropriate rough SDEs, where the initial time and the initial state play the role of parameters.

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Speaker:
Benjamin Robinson

Title:
Bicausal optimal transport for SDEs with irregular coefficients

Abstract:
Many natural phenomena that exhibit randomness can be modelled by stochastic differential equations (SDEs), often having less regularity than the classical case of SDEs with Lipschitz coefficients. In such settings, we are interested in quantifying model uncertainty and the impact of model choice on the value of stochastic optimisation problems. To this end, we seek an appropriate notion of distance on the space of models. In particular, we study the adapted Wasserstein distance between the laws of SDEs. This is a special case of a bicausal optimal transport problem, in which the classical optimal transport problem is constrained to respect the flow of information inherent in stochastic processes.

Under minimal regularity assumptions on the coefficients, we show that the value of the bicausal optimal transport problem between the laws of one-dimensional SDEs is attained by the synchronous coupling. This is the coupling induced by taking a common Brownian motion as the driving noise for each SDE. Our proof is based on a discretisation method, exploiting monotonicity properties of the resulting discrete-time processes. A key tool in our work is a transformation-based semi-implicit Euler—Maruyama scheme for SDEs whose drift coefficient may have discontinuities and exponential growth. We prove the first strong existence and uniqueness result for such SDEs, and we obtain strong convergence rates for the implicit scheme. Moreover, our results provide a method for efficient computation of the adapted Wasserstein distance.

This is joint work with Michaela Szölgyenyi (University of Klagenfurt).

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Speaker:
Thomas Wagenhofer

Title:
Weak Error Rates for Local Stochastic Volatility Models

Abstract:
We study the error rates of discretised local volatility models. To achieve a good fitting of these models to real-world data one ends up with a singular McKean Vlasov SDE, where the coefficients depend on the conditional expectation given the solution of the

equation. Aside from special cases existence and uniqueness-theory for the resulting McKean Vlasov equation is (mostly) an open problem. Nevertheless the one-dimensional distributions can be characterized easily via so-called Markovian mimicking. Our analysis focuses on the Euler Scheme for the McKean-Vlasov SDE in combination with a particle method. The main result then provides quantitative error bounds based on all relevant parameters.

This is joint work with Peter Friz.

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Speaker:
Paul Hager

Title:
Advancing Optimal Stochastic Control with Signatures

Abstract:
We propose to parameterize open loop controls in stochastic optimal control problems via suitable classes of functionals depending on the driver's path signature, a concept adopted from rough path integration theory. We rigorously prove that these controls are dense in the class of progressively measurable controls and use rough path methods to establish suitable conditions for stability of the controlled dynamics and target functional. These results pave the way for Monte Carlo methods to stochastic optimal control for generic target functionals and dynamics. We discuss the rather versatile numerical algorithms for computing approximately optimal controls and verify their accurateness in benchmark problems from Mathematical Finance.

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Speaker:
Oleg Butkovsky

Title:
Weak uniqueness for singular stochastic equations

Abstract:
We put forward a new method for proving weak uniqueness of stochastic equations with singular drifts driven by a non-Markov or infinite-dimensional noise. We apply our method to study stochastic heat equation driven by Gaussian space-time white noise and SDEs driven by fractional Brownian motion, where the drift b is a generalized function in the Besov space of regularity alpha. Well-known pathwise uniqueness results for these equations do not cover the entire range of the parameter alpha, for which weak existence holds. What happens in the range where weak existence holds but pathwise uniqueness is unknown has been an open problem. We settle this problem and show that for SHE weak uniqueness holds for alpha>−3/2, and for SDE it holds for alpha>1/2−1/(2H); thus, in both cases, it holds in the entire desired range of values of alpha. This extends seminal results of

Catellier and Gubinelli (2016) and Gyöngy and Pardoux (1993) to the weak well-posedness setting. To establish these results, we develop a new strategy, combining ideas from ergodic theory (generalized couplings of Hairer-Mattingly-Kulik-Scheutzow) with stochastic sewing of Lê.

This is joint work with Leonid Mytnik.

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Speaker:
Franziska Bielert

Title:
Conditional Stochastic Optimal Control

Abstract:
We consider a stochastic optimal control problem with anticipative controls. This leads to a random path-dependent value function. We provide an HJB equation using the functional Itô formula. It includes a novel type of condition on the path-dependent derivatives of the value function.

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Speaker:
Verena Schwarz

Title:
Milstein scheme for jump-diffusion McKean-Vlasov SDEs

Abstract:
In this talk we derive a general Milstein-type scheme for the approximation of the jump-diffusion McKean-Vlasov SDEs and the associated system of interacting particles and provide a strong error analysis. Further we discuss simplifications of the Milstein-type scheme for a direct approximation of the jump-diffusion McKean-Vlasov SDEs and numerical implementation.

This is joint work with Sani Biswas, Chaman Kumar and Christoph Reisinger.

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Speaker:
Peter Friz

Title: Signatures, developments, smooth rough paths and triviality of paths with polynomial log-signature

Abstract: N/A