## Program Stream Discrete Optimization

*Talks I:*
Martin Skutella: **Unsplittable Multiflows in Series-Parallel Networks**
Daniel Brosch: **New Lower Bounds on Crossing Numbers of $K_{m,n}$**
Jutta Rath (short talk): **Associated Primes and Graphs**

*Plenary Discrete Optimization:*
Max Klimm: **Information Design for Congested Networks**

*Talks II:*
Svenja M. Griesbach: **Deterministic Impartial Selection with Weights**
Lea Strubberg: **Recovering Potential-based Flow Networks during Operations and under Uncertainty**
Diane Puges: **Sums of Squares for Infinite Trees**
Dunja Pucher (short talk): **On Upper Bounds for the Maximum $k$-colorable Subgraph Problem**

*Talks III:*
Moritz Grillo: **Complexity of Deciding Injectivity and Surjectivity of ReLU Neural Networks**
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## Abstracts

Speaker (Plenary):
Max Klimm

Title:
Information Design for Congested Networks

Abstract:
We consider a novel approach to improve traffic networks that is rooted in the inherent uncertainty of travel times or demands. As this data is subject to stochastic uncertainty resulting from various parameters such as weather conditions, occurrences of road works, or traffic accidents. Large mobility services have an informational advantage over single network users as they can learn traffic conditions from data. A benevolent mobility service may use this informational advantage to steer the traffic equilibrium in a favorable direction. The resulting optimization problem is a task commonly referred to as signaling. We tightly characterize the class of single-commodity networks, in which full information revelation is always an optimal signaling strategy. Moreover, we construct algorithms computing the optimal signaling scheme whose complexity is polynomial in the number of

support vectors that arise in the equilibrium. While this number may be exponential in worst-case instances, it is relatively small in realistic instances. Using a cell decomposition technique, we obtain a polynomial-time algorithm for multi-commodity parallel edge networks with a constant number of commodities, even when we have a constant number of different states of nature.

This is joint work with Svenja M. Griesbach, Marton Hoefer, Tim Koglin, Philipp Warode, and Theresa Ziemke

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Speaker:
Daniel Brosch

Title:
New Lower Bounds on Crossing Numbers of $K_{m,n}$

In this talk, we use semidefinite programming and representation theory to compute new lower bounds on the crossing number of the complete bipartite graph $K_{m,n}$, extending a method from de Klerk et al. [SIAM J. Discrete Math. 20 (2006), 189--202] and the subsequent reduction by De Klerk, Pasechnik and Schrijver [Math. Prog. Ser. A and B, 109 (2007) 613--624].

We exploit the full symmetry of the problem by developing a block-diagonalization of the underlying matrix algebra, and use it to improve bounds on several concrete instances. Our results imply that $\mathrm{cr}(K_{10,n}) \geq  4.87057 n^2 - 10n$, $\mathrm{cr}(K_{11,n}) \geq 5.99939 n^2-12.5n$, $\mathrm{cr}(K_{12,n}) \geq 7.25579 n^2 - 15n$, $\mathrm{cr}(K_{13,n}) \geq 8.65675 n^2-18n$ for all~$n$. The latter three bounds are computed using a new relaxation of the original semidefinite programming bound, by only requiring one small matrix block to be positive semidefinite. Our lower bound on $K_{13,n}$ implies that for each fixed $m \geq 13$, $\lim_{n \to \infty} \text{cr}(K_{m,n})/Z(m,n) \geq 0.8878 m/(m-1)$. Here $Z(m,n)$ is the Zarankiewicz number: the conjectured crossing number of $K_{m,n}$.

This is joint work with Sven Polak.

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Speaker:
Svenja M. Griesbach

Title:
Deterministic Impartial Selection with Weights

Abstract:
In the impartial selection problem, a subset of agents up to a fixed size $k$ among a group of $n$ is to be chosen based on votes cast by the agents themselves. A selection mechanism

is impartial if no agent can influence its own chance of being selected by changing its vote. It is $\alpha$-optimal if, for every instance, the ratio between the votes received by the selected subset is at least a fraction of $\alpha$ of the votes received by the subset of size $k$ with the highest number of votes. We study deterministic impartial mechanisms in a more general setting with arbitrarily weighted votes and provide the first approximation guarantee, roughly $1/\lceil 2n/k\rceil$. When the number of agents to select is large enough compared to the total number of agents, this yields an improvement on the previously best known approximation ratio of $1/k$ for the unweighted setting. We further show that our mechanism can be adapted to the impartial assignment problem, in which multiple sets of up to $k$ agents are to be selected, with a loss in the approximation ratio of $1/2$.

This is joint work with Javier Cembrano and Maximilian Stahlberg.

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Speaker:
Moritz Grillo

Title:
Complexity of Deciding Injectivity and Surjectivity of ReLU Neural Networks

Abstract:
Neural networks with ReLU activation play a key role in modern machine learning. In view of safety-critical applications, the verification of trained networks is of great importance and necessitates a thorough understanding of essential properties of the function computed by a ReLU network, including characteristics like injectivity and surjectivity. Recently, Puthawala et al. [JMLR 2022] came up with a characterization for injectivity of a ReLU layer, which implies an exponential-time algorithm. However, the exact computational complexity of deciding injectivity remained open.
We answer this question by proving coNP-completeness of deciding injectivity of a ReLU layer. On the positive side, we present a parameterized
algorithm which yields fixed-parameter tractability of the problem with respect to the input dimension. In addition, we also characterize surjectivity for two-layer ReLU networks with one-dimensional output. Finally, we reveal interesting connections to computational convexity by formulating the surjectivity problem as a zonotope containment problem.

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Speaker:
Dunja Pucher

Title:
On Upper Bounds for the Maximum $k$-colorable Subgraph Problem

Abstract:

In this short talk we present bounds on the maximum $k$-colorable subgraph problems from Olga Kuryatnikova, Renata Sotirov and Juan Vera and ideas on how to improve on this bounds. Furthermore, we discuss a conjecture relating one of these bounds to $\theta_+$.

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Speaker:
Diane Puges

Title:
Sums of Squares for Infinite Trees

Abstract:
Rooted leaf-labeled binary trees are objects introduced, and widely used, in phylogenetics. The question of their inducibility, in particular, is a central problem in this field. We extend Razborov's flag-algebra theory to this setting. Flag-algebras are a very powerful tool of extremal combinatorics, that have already been successfully used to tackle a large variety of problems in combinatorics and geometry. This allows the application of sums-of-squares and moment techniques to extremal questions about trees. We introduce and apply these methods, that enable us to improve existing bounds and obtain many new ones on the inducibility of trees. Moreover, we are able to obtain the first approximations of density profiles of rooted leaf-labeled binary trees.

This is joint work with Daniel Brosch.

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Speaker:
Jutta Rath

Title:
Associated Primes and Graphs

Abstract:
To every ideal in a ring one can associate a unique set of prime ideals, its so-called associated primes.
In many settings, these primes can be interpreted as structure revealing building blocks of $I$. The associated primes of an ideal in the integers correspond to the prime divisors of the generator of the ideal;
the associated primes of the inducing ideal of an algebraic variety correspond to the its irreducible components; for the edge ideal of a finite simple graph, the associated primes are precisely the prime ideals generated by the minimal vertex covers of the graph. This talk focuses on the connection between associated primes and graph theory.

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Speaker:
Jan Schwiddessen:

Title:
A Low-rank SDP Approach for Semi-Supervised Support Vector Machines

Abstract:
Support vector machines (SVMs) are well-studied supervised learning models for binary classification. They have proven to be powerful machine learning techniques, and efficient algorithms exist to solve the underlying optimization problems. However, most data in real life is unclassified, leading to semi-supervised support vector machines (S3VMs) instead. State-of-the-art MIP and MINLP solvers can only solve small S3VM instances to optimality. In the first part of this talk, we present a new branch-and-cut approach for S3VMs using semidefinite programming (SDP) relaxations. We first apply optimality-based bound tightening to bound the feasible set. Then, at each branch-and-bound node, we strengthen the SDPs by adding RLT cuts and applying bound tightening techniques. We provide numerical results showing that our approach is capable of producing very small duality gaps for real-world instances. In the second part of this talk, we present a new first-order method for solving the basic SDP relaxation of S3VMs models. It is based on the Burer-Monteiro factorization, exploits low-rank solutions of S3VM models, and implements a simple but very efficient coordinate descent method.

This is joint work with Veronica Piccialli and Antonio M. Sudoso.

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Speaker:
Melanie Siebenhofer

Title:
Solution Approaches for a Fractional Combinatorial Problem

Abstract:
In this blackboard talk we will present three different solution approaches to compute the edge expansion of a graph. The first approach, called split & bound, splits the problem into several k-bisection problems. The second approach is based on parametrized optimization and follows an approach of Dinkelbach. Finally, the third method, a convexification approach, reformulates the problem as copositive program which then can be relaxed as a double non-negative problem.

This is based on joint work with Akshay Gupte, Timotej Hrga and Angelika Wiegele

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Speaker:
Martin Skutella

Title:
Unsplittable Multiflows in Series-Parallel Networks

Abstract:
We study the unsplittable multiflow problem where the demand of every commodity must be routed along a single path from its source to its sink node. As our main result we prove that, in series-parallel digraphs, a given multiflow can always be written as a convex combination of unsplittable multiflows whose total flow values on any arc deviate from the given flow by less than the maximum demand of any commodity; moreover, such a convex combination can be efficiently computed. Our result implies that a 25-year-old conjecture of Goemans for single-source unsplittable flows, as well as a stronger recent conjecture of Morell and Skutella, both hold for series-parallel digraphs, even in the more general setting where commodities do not necessarily share a common source node. So far, no non-trivial class of digraphs has been known for which Goemans' conjecture holds.

Our result implies the following integrality property for multiflows in series-parallel digraphs: For integer demands
and integer arc capacities, the existence of a feasible (fractional) multiflow implies the existence of a feasible
integer multiflow. In this context, it is interesting to observe that, for series-parallel digraphs, the cut
condition does not imply the existence of a feasible (fractional) multiflow. In particular, the integrality property
cannot be obtained via the classical result of Nagamochi and Ibaraki on the existence of integer multiflows.

This is joint work with Mohammed Majthoub Almoghrabi and Philipp Warode.

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Speaker:
Lea Strubberg

Title:
Recovering Potential-based Flow Networks during Operations and under Uncertainty

Abstract:
In potential-based flow networks, the flow along an arc depends on the potential difference between its connecting nodes. A prime example of potential-based flow networks are gas networks, with arcs representing pipes and potentials correlating to the pressure at the nodes. Typically, potentials are measured and controlled at the entry and exit nodes, also referred to as terminal nodes.
After Klimm et. al (2021) studied the reduction of potential-based flow networks to smaller, equivalent networks with the same behavior at the terminal nodes, we analyze the question of whether a network can be recovered from a limited amount of measurement data.

In this talk, we will first recover tree-networks from their effective resistances. The effective resistance between two terminal nodes is given by their potential difference when sending one unit of flow from one of the terminals to the other. However, a practical implementation of this procedure necessitates interrupting current network operations to determine the effective resistance of each terminal pair.
Consequently, we introduce an alternative method to reconstruct the network during ongoing network operations, allowing only minor deviation from a given demand scenario. Employing a sensitivity analysis of the potential differences under different demand deviations, we demonstrate how to recover a tree network up to equivalence. Furthermore, we show that this method can be applied even if there exists a leak in our network, where a leak acts like a terminal node which we cannot control.

This is joint work with Max Klimm, Marc Pfetsch, Rico Raber, and Martin Skutella.